Grade 8 Surface Area of cylinder

8.SS.3	
Determine the surface area of • right rectangular prisms • right triangular prisms • right cylinders to solve problems.	1. Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a 3 D object. (ONLY FOR CYLINDER) 2. Identify all the faces of a prism, including right rectangular and right triangular prisms. (for CYLINDER) 3. Describe and apply strategies for determining the surface area of a right rectangular or right triangular prism. (NOT DEVELOPED) 4. Describe and apply strategies for determining the surface area of a right cylinder. (DEVELOPED) 5. Solve a problem involving surface area. (for CYLINDER)

Clarification of the outcome:

- ✦ The outcome is unpacked into three parts: (1) surface area of right rectangular prism, (2) surface area of right triangular prism, and (3) surface area of right cylinder. The reason is that each object has a distinct surface area formula that is not well related to the other formulas.
- ✦ The focus here is the surface area of a right cylinder (technical language for a can). Surface area is equivalent to the area of the skin of the can (the curved side and the two circles that compose it). Another way to say this is that surface area is the area of the faces of an object, in this case the area of the curved side + the area of the two circlers. [Note: A face can be curved.]

Required close-to-at-hand prior knowledge:

- ✤ Understand area as a count of identical squares that cover a surface and that these squares are measured by square cm (cm^2) in the metric system.
- ✤ Understand surface area in relation to right and triangular prisms. This assumes these parts of outcome 8.SS.3 have already been developed (the wisest sequencing because those surface areas are conceptually simpler than that of a right cylinder).
- ✤ Understand the area of a circle and its formula: $\pi * Radius^2$
- ✤ Understand the circumference of a circle and its formula: $2\pi * Radius$
- ✤ Understand the area of a rectangle and its formula (**base x height**)

SET SCENE stage

The problem task to present to students:

Organize students into groups. Provide each group with a small to medium-sized plastic can (right cylinder). Tell them that the can is to be decorated by covering all faces of it with coloured paper. The problem is to determine how much paper is needed. Ask the groups to solve the problem any way they want.

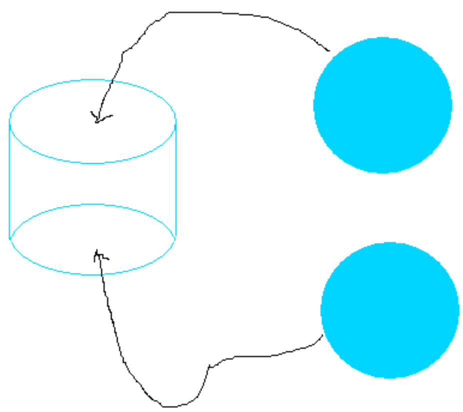
Comments:

The main purpose of the task is to engage students in preliminary thinking about the surface area of a cylinder (can) without involving jargon or formal concepts.

DEVELOP stage

Activity 1: Revisits SET SCENE, and addresses achievement indicators 1, 2, 3, and 5.

- Ask selected groups to present their strategy for determining the amount of paper needed to cover the can. Accept all responses.
- Ask students if the problem involved finding the surface area of a can (this assumes they understand the concept of surface area because it was already developed for a prism). Ensure they realize that the amount of paper to cover corresponds to the surface area of the can (ignoring waste when working with the paper). Ask them to identify the faces that make up the can and how to use that information to determine surface area. Ensure they realize the surface area involves finding the area of two circles and a curved side.



- Discuss how to find the area of the two circles. [Note: This part should be straight forward as long as students understand the area of a circle and its formula.] Place the result for the surface area obtained to this point on the white board.

Surface area of cylinder is:

area of top circle + area of bottom circle + area of curved side

This becomes:

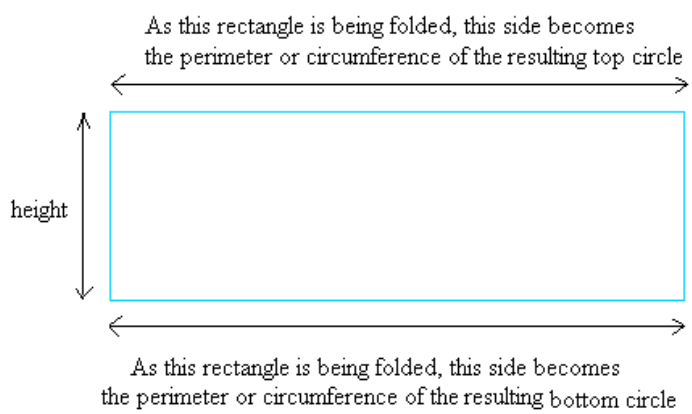
$PI*R^2 + PI*R^2$ + area of curved side

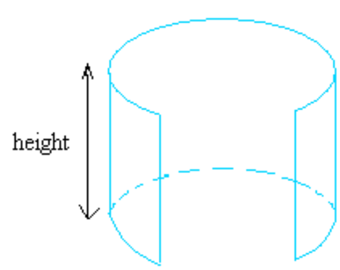
Activity 2: Addresses achievement indicators 1, 2, 3, and 5.

- Tell students that the remaining challenge is to figure out the area of the curved side. Ask each group to unwrap the can by: (1) placing it on a sheet of paper, drawing a line on the paper where the can touches the paper, (2) marking this line on the can, (3) rolling the can along the sheet of paper until the mark on the can touches the paper again, (4) drawing a line on the paper where the marked line touches the paper again, and (5) making a rectangle using the two drawn lines as the ends. [Note: Refer to the following website.]

  Surface area of a **cylinder -** Math Open Reference

- Provide a rectangular sheet of paper and ask students to use it to make a cylinder (a can). Ensure they realize that joining the ends creates a cylinder.
- Ask students to think about the relationship between the drawn rectangle and the rectangular sheet of paper and the curved side of the can. Ask students if the area of the rectangle is the same as the area of the curved side. Ensure they realize this. Discuss the area of rectangle as base x height. Suggest that base and height are the two dimensions/ side lengths of the rectangle. Ask them what one dimension of the drawn rectangle is. Ensure they realize it is the height of the right cylinder.
- Ask them what the other dimension is. As an assist, ask them to measure the circumference of the top circle of the can by laying a string around it. Ask them to compare the resulting length of the string to the length of one of the sides of the drawn rectangle (the one obtained by unwrapping the can). Discuss why the circumference of the circle is one of the dimensions of the rectangle. Ask them how to calculate the circumference. Ensure they realize it is: 2 * PI * R.





- Revisit what was written on the whiteboard. Ask students to fill in the missing part. Ensure students understand what each part refers to for the cylinder. Once that is clear, ask them to simplify the formula. Assist them to express the formula in the shortest way. Expect as shown below.

Surface area of cylinder is:

AREA OF TOP CIRCLE + AREA OF BOTTOM CIRCLE + AREA OF CURVED SIDE

PI * $radius^2$ + PI * $radius^2$ + AREA OF CURVED SIDE

$$\pi * radius^2 + \pi * radius^2 + 2 * \pi * radius * height$$

$$2 * \pi * radius^2 + 2 * \pi * radius * height$$

Activity 3: Addresses achievement indicators 1, 2, 3, and 5, and practice.

- Organize students into pairs. Student #1 asks student #2 to explain how the formula for the surface area of a right cylinder was figured out.
- Student #2 asks student #1 to explain what surface area means and to make up a simple problem about the surface area of a right cylinder. Both students solve the problem and compare results. The teacher helps as appropriate.

Activity 4: Addresses achievement indicators 4 and 5, and practice.

- Organize students into pairs. Provide about five problems concerning the surface area of a right cylinder. Each pair solves the problems and discusses whether each solution is correct. [Note: Some problems should be basic and some should be more difficult. See below for an example of each.]

  BASIC:

  Find the surface area of a right cylinder having a radius of 4 cm, and a height of 3 cm. Use PI = 3.1

  MORE DIFFICULT:

  As a project in tech class, the students are making aluminum flower pots shaped like right cylinders. Each pot has a bottom but no top. If the pots are 25 cm in height and have a diameter of 40 cm, how many square cm of aluminum are needed for six such pots, assuming no wastage of aluminum? How would wastage change your answer?

- Ask selected pairs to present the solution to the problems.

Activity 5: Assessment of teaching.

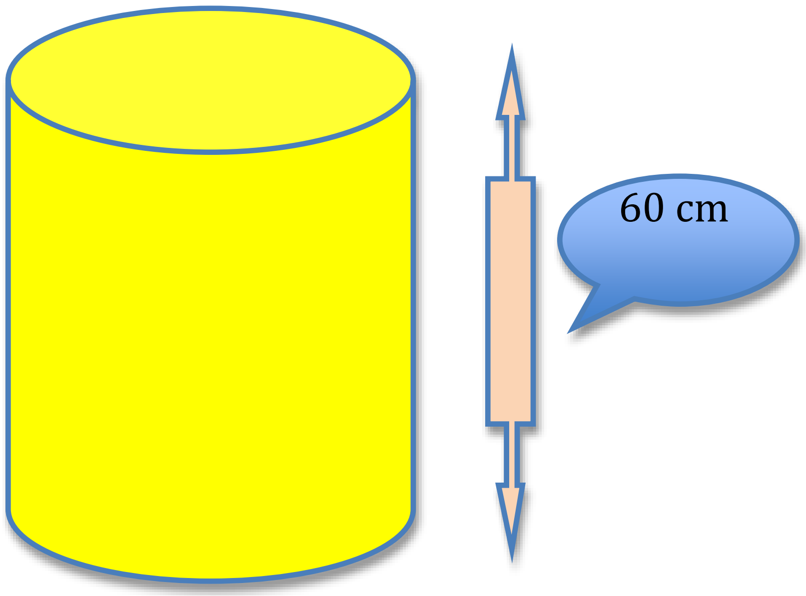
- Provide two problems about the surface area of a right cylinder: one basic and one somewhat more difficult. DO NOT provide the formula. Students should be able to remember it by now. Samples are shown below.

BASIC:

Find the surface area of a right cylinder having a radius of 5 cm, and a height of 10 cm. Use PI = 3

MORE DIFFICULT:

Mark wants to find the surface area of a large right cylindrical bucket that has no top. Mark notices that on piece of information is missing.



(a) What information is missing? Estimate a value for it. ____________

(b) What is the surface area of the bucket if the top is not included?

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheets.

Question 1.

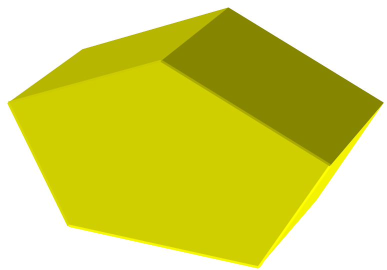
For each object:

(a) Identify and label the **<u>different</u>** faces as one of A, B, C, . . . on the object.

(b) Write a faces formula for figuring out the surface area of the object that involves the labelled faces. For example a formula might be: A + B + 2C.

Object #1

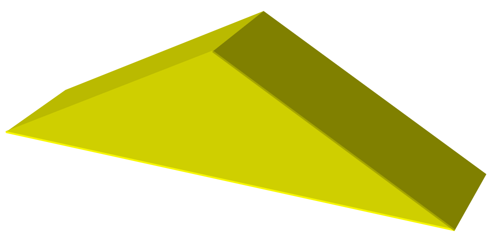
(a)



(b) Faces formula for surface area = ______________________________

Object #2

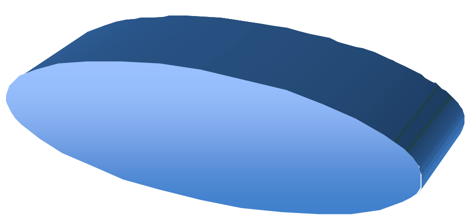
(a)



(b) Faces formula for surface area = ______________________________

Object #3

(a)



(b) Faces formula for surface area = ______________________________

Question 2.

Find the surface area of each right cylinder. Use PI = 3.1

a) Height = 12 cm, radius = 5 cm

b) Height = 30 cm, diameter = 8 cm

Question 3.

John is considering buying a right cylindrical water container. The container needs to be insulated on the top, bottom, and side. The store has two containers to choose from.

Container #1 has a height of 120 cm and a radius of 30 cm.

Container #2 has a height of 100 cm and a radius of 35 cm.

If John wants to use the least amount of insulation (assuming constant thickness of insulation), which container should he buy?

Explain your thinking and show your work. [Use PI = 3.1.]

MAINTAIN stage

Mini-task example

Every so often:

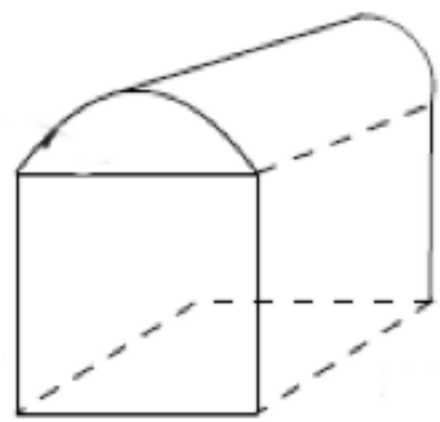
- Present a drawing of a right cylinder, with measurements given. Ask students to determine the surface area. Use PI = 3.

Rich-task example #1

Present this problem.

A storage shed has a box-shaped base and a roof in the shape of half of a right cylinder. The front of the box is a square having side length 3 m. The length of the box is 4 metres. The distance from the top of the box to the peak of the half cylinder is 1 metre.

The shed needs to be painted (roof and all sides). One litre of paint covers 20 square metres. How many litres of paint must be bought? Use PI = 3.14



Rich-task example #2

Present this problem.

The surface area of large right cylinder to be used for an insulated hot water heater (insulated on top, bottom, and side) should be as close to 36 000 square cm as possible because of restrictions placed on insulation material.

What are the height and radius of a right cylinder that satisfies that condition? Use PI = 3.1 Explain your thinking and show your work.

Comments

These are a rich-tasks because each is a complex problem that integrates surface area with other matters.